Response Function of Hot Nuclear Matter

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Abstract: We investigate the response function of hot nuclear matter to a small isovector external field using a simplified Skyrme interaction reproducing the value of the symmetry energy coefficient. We consider values of the momentum transfer corresponding to the dipole oscillation in heavy nuclei. We find that while at zero temperature the particle hole interaction is almost repulsive enough to have a sharp (zero sound type) collective oscillation, such is no longer the case at temperatures of a few MeV. As a result a broadening of the dipole resonance occurs, leading to its quasi disappearence by the time the temperature reaches 5 MeV. The sensivity of the temperature evolution of the width when modifying the residual interaction strength is also examined.

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A large amount of data has been accumulated over the years concerning the properties of giant resonances in hot nuclei [1, 2] and the evolution of their energy and width as temperature increases. To a good accuracy the energy of giant dipole resonance is found to be independent of the excitation energy while its width is found first to increase at low excitations energies with a saturation at higher energies [3]. On the theoretical side the energy of the giant dipole resonance is well understood already at the level of the random phase approximation (RPA). Concerning the increase of the width with T however, the situation is still unsettled. Indeed although there are various mechanisms contributing to this increase such as shape fluctuations or angular momentum effects [4, 5] their total contribution does not seem to account for the observed data. Furthermore there are also dynamical effects inhibiting this increase [6]. It is worthwhile noting that in some of the early RPA calculations using oscillator functions [7] a rapid increase of the width was found. This result was however not confirmed in the self-consistent RPA calculations including continuum states performed by Sagawa and Bertsch [8]. Recently the early RPA calculations have been reexamined by Nicole Vinh Mau [9] who concluded that one important ingredient in these calculations was the appearence of new particle-hole configurations as temperature rises, an effect to which more attention should be given when trying to explain the broadening of the giant dipole resonance. Why are these effects found to be nearly negligible in the calculations of Sagawa and Bertsch? The aim of the present letter is to examine this question in the framework of a self consistent calculation of the response function of nuclear matter using a schematic Skyrme force. The advantage of this calculation is that while incorporating a full self-consistency it is still tractable and leads to transparent formulae. In particular these formulae show that while at zero temperature the isovector particle -hole residual interaction is almost repulsive enough to have a well developped zero sound, such is no longer the case at temperatures of a few MeV, leading in general to a rapid weakenig of collective effects.

In the case of nuclear matter single particle states are labeled by their wave number $\mathbf{k}$, their spin σ and their isospin τ. We wish to calculate the response to an external field of the form:

$$V_{ext} = \epsilon \tau_3 e^{-i\mathbf{q}.\mathbf{r}} e^{-i(\omega+i\eta)t}. \tag{1}$$

where τ_3 is the third component of the isospin operator and η a vanishingly small positive number corresponding to an adiabatic switching of the external field. For this purpose we use mean field theory. In this case the evolution of the one-body density matrix ρ is determined by the time dependent Hartree-Fock equation:

$$i\partial_t \rho = [W + V_{ext}, \rho], \tag{2}$$

where W is the mean field hamiltonian. Let us consider for simplicity the case of a schematic Skyrme force:

$$v = t_0(1 + x_0 P_\sigma)\delta(\mathbf{r}_1 - \mathbf{r}_2) + t_3 \delta(\mathbf{r}_1 - \mathbf{r}_2)\delta(\mathbf{r}_2 - \mathbf{r}_3). \tag{3}$$

In this case we have

$$W = \frac{p^2}{2m} + U(r), \tag{4}$$

where U is a local potential, given in the case of neutrons by:

$$U_n(\mathbf{r}, t) = \frac{t_0}{2}\{2\rho(\mathbf{r}, t) - \rho_n(\mathbf{r}, t)\} + \frac{t_3}{4}\{\rho^2(\mathbf{r}, t) - \rho_n^2(\mathbf{r}, t)\}, \tag{5}$$

with a similar expression for protons. In the previous equation $\rho(\mathbf{r}, t)$ is the density distribution $< \mathbf{r}|\rho|\mathbf{r} >$. An identical equation would be obtained with a density dependent version of the Skyrme force [10] and thesubsequent developpement would be unmodified. For a small enough external field it is legitimate to linearize the mean field evolution equation (2). This procedure leads to the following approximate equation for the difference $\delta\rho = \rho_n - \rho_p$ of the neutron and proton density matrices:

$$\begin{aligned} i\partial_t < \mathbf{k}|\delta\rho|\mathbf{k}' >= \quad & \{\epsilon(\mathbf{k}) - \epsilon(\mathbf{k}')\} < \mathbf{k}|\delta\rho|\mathbf{k}' > + \{f(\mathbf{k}') - f(\mathbf{k})\} < \mathbf{k}|(U_n - U_p)|\mathbf{k}' > \\ & + 2\epsilon\{f(\mathbf{k}') - f(\mathbf{k})\}\delta(\mathbf{k}' - \mathbf{k} - \mathbf{q})e^{-i(\omega + i\eta)t}. \end{aligned} \tag{6}$$

In this equation $\epsilon(\mathbf{k}) = \hbar^2 k^2/2m$ is the energy of the single particle state with wave number $\mathbf{k}$ and

$$f(\mathbf{k}) = 1/\{1 + e^{\beta(\epsilon(\mathbf{k}) - \mu)}\}, \tag{7}$$

is the corresponding occupation number. Note that in the linear response approximation we have from equation (5):

$$U_n(\mathbf{r}, t) - U_p(\mathbf{r}, t) = 2V_0 \delta\rho(\mathbf{r}, t) \tag{8}$$

where

$$V_0 = -\frac{t_0}{2}(x_0 + \frac{1}{2}) - \frac{t_3}{8}\rho_0, \tag{9}$$

ρ_0 being the saturation density of nuclear matter. Note that V_0 is related to the symmetry energy coefficient a_τ of nuclear matter via the relation:

$$a_\tau = \frac{1}{3}T_F + \frac{1}{2}V_0\rho_0. \tag{10}$$

where T_F is the kinetic energy at the Fermi surface. The above equations suggest to look for a solution of the evolution equation (6) in the form:

$$< \mathbf{k}|\delta\rho(t)|\mathbf{k'} >= \delta(\mathbf{k'} - \mathbf{k} - \mathbf{q}) < \mathbf{k}|\delta\rho(t=0)|\mathbf{k}+\mathbf{q} > e^{-i(\omega+i\eta)t}. \tag{11}$$

For this Ansatz the change in the density distribution has the following structure:

$$\delta\rho(\mathbf{r},t) = \alpha e^{-i\mathbf{q}.\mathbf{r}}e^{-i(\omega+i\eta)t}, \tag{12}$$

where:

$$\alpha = \sum_{\mathbf{k}} < \mathbf{k}|\delta\rho(t=0)|\mathbf{k}+\mathbf{q} > . \tag{13}$$

The change between the neutron and proton potentials is thus found from equation (8) to be:

$$< \mathbf{k}|(U_n - U_p)|\mathbf{k'} >= 2V_0\alpha\delta(\mathbf{k'} - \mathbf{k} + \mathbf{q})e^{-i(\omega+i\eta)t}. \tag{14}$$

Inserting this expression into the linearized evolution equation for the density matrix (6) we find that our Ansatz is indeed a solution provided that:

$$< \mathbf{k}|\delta\rho(0)|\mathbf{k}+\mathbf{q} >= \frac{(2V_0\alpha + 2\epsilon)\{f(\mathbf{k}) - f(\mathbf{k}+\mathbf{q})\}}{\omega + i\eta - \epsilon(\mathbf{k}) + \epsilon(\mathbf{k}+\mathbf{q})}. \tag{15}$$

Returning to equation (13) we obtain a linear relation determining the value of α:

$$\alpha = (V_0\alpha + \epsilon)\Pi_{0R}(\omega,\mathbf{q}), \tag{16}$$

In this equation Π_{0R} is the unperturbed retarded response function defined by:

$$\Pi_{0R}(\omega,\mathbf{q}) = \frac{2}{(2\pi)^3}\int d\mathbf{k}\frac{f(\mathbf{k}+\mathbf{q}) - f(\mathbf{k})}{\omega + i\eta - \epsilon(\mathbf{k}) + \epsilon(\mathbf{k}+\mathbf{q})}, \tag{17}$$

Solving equation (16) for α we obtain the following expression for the retarded response function Π_R in the RPA approximation:

$$\Pi_R(\omega, \mathbf{q}) \equiv \frac{\alpha}{\epsilon} = \frac{\Pi_{0R}(\omega, \mathbf{q})}{1 - V_0 \Pi_{0R}(\omega, \mathbf{q})}. \tag{18}$$

The result we have derived is valid only in the case of a simplified Skyrme force. For other interactions the quantity V_0 would not be a c-number. The case of a full Skyrme force was worked out at zero temperature by Garcia-Recio et al [11] who still obtained a closed expression for the response function at the cost of introducing additional unperturbed functions. For a given momentum $\mathbf{q}$ we find that there is a resonant response when the frequency ω corresponds to a zero of the denominator, i.e., when:

$$1 = V_0 \Pi_{0R}(\omega, \mathbf{q}). \tag{19}$$

The real part of ω determines the energy of the collective mode while $\Im m(\omega)$ determines its life time [12]. The importance of the resonance condition (19) is also seen when looking at the distribution of strength per unit volume $S(\omega)$ for the operator $\exp(i\mathbf{q}.\mathbf{r})$. Indeed it is given by $-\Im m \Pi_R/\pi$ i.e. [12]

$$S(\omega) = -\frac{1}{\pi} \frac{\Im m \Pi_{0R}(\omega, q)}{(1 - V_0 \Re e \Pi_{0R})^2 + (V_0 \Im m \Pi_{0R})^2}. \tag{20}$$

For symmetric nuclear matter this function enjoys the following sum rule

$$\int_0^\infty \omega S(\omega) d\omega = \frac{\hbar^2}{2m} \rho_0 q^2. \tag{21}$$

Let us now try to apply the previous formulae to the case of finite nuclei. For this purpose a usefull guide is the Steinwedel and Jenssen model [10]. In this model neutrons and protons oscillate inside a sphere of radius R according to the formula:

$$\rho_n(\mathbf{r}, t) - \rho_p(\mathbf{r}, t) = \varepsilon \sin(\mathbf{q}.\mathbf{r}) \exp(i\omega t), \tag{22}$$

the total density remaining equal to the saturation density ρ_0 and the wavenumber q being given by

$$q = \frac{\pi}{2R}. \tag{23}$$

In order to describe the dipole resonance in finite nuclei this model thus suggests to look at the strength function $S(\omega, q)$ calculated at a momentum transfer $q = \pi/2R$, where R is the nuclear radius. In the following we shall focus on the nucleus lead-208 for which we take $R = 6.7fm$ and $q = .23fm^{-1}$. We use the following values of the parameters: $t_0 = -983.4$ MeV $\times$ fm^3, $t_3 = 13106$ MeV $\times$ fm^6 and $x_0 = .48$. These values were fitted to reproduce the binding energy (E/A= -16MeV), the saturation density($\rho_0 = 0.17$ fm^{-3}) and the symmetry energy($a_\tau = 30$ MeV) of nuclear matter. For these values $V_0 = 203MeVfm^3$.

To calculate S(ω) we still need the expression of the unperturbed Lindhard function. The imaginary part can be expressed in terms of elementary functions as [13]:

$$\Im m\Pi_{0R}(\omega, q) = -\frac{kT}{8\pi q}\left(\frac{2m}{\hbar^2}\right)^2 \log\frac{1+e^{\beta(A+\omega/2)}}{1+e^{\beta(A-\omega/2)}} \tag{24}$$

where $\beta = 1/kT$ while

$$A = \mu - \frac{\omega^2}{4\epsilon(q)} - \frac{\epsilon(q)}{4}. \tag{25}$$

In this equation μ is the chemical potential and $\epsilon(q)$ the single particle energy of the state with momentum q. The real part can be expressed as:

$$\Re e\Pi_{0R} = -\int F(k)df(k) \tag{26}$$

where $f(k)$ is the occupation number while:

$$F(k) = \frac{1}{2\pi^2}\frac{mk}{\hbar^2}\left\{-1 + \frac{k}{2q}\left[\phi(\frac{mw}{\hbar kq} + \frac{q}{2k}) - \phi(\frac{mw}{\hbar kq} - \frac{q}{2k})\right]\right\} \tag{27}$$

with:

$$\phi(x) = (1-x)(1+x)\log|\frac{x-1}{x+1}|. \tag{28}$$

Note that at zero temperature we have $df(k) = -\delta(k-k_F)dk$ so that $F(k)$ is just the zero temperature value of the real part of Π_{0R}. This implies that equation (26) is well suited for a numerical integration centered around the value $k = \sqrt{2m\mu/\hbar^2}$.

The real part of the Lindhard function is graphed on figure 1 as a function of the frequency ω for $q = .23fm^{-1}$ and several values of the temperature T=0,2,4,6 MeV. It can be seen that at T=0 the quantity $\Re e\Pi_{0R}$ has a maximum near $\omega = 13MeV$ which is not quite sufficient to

produce a zero in the quantity $1 - V_0 \Re e \Pi_{0R}$. Such a zero would correspond to a sharp collective oscillation analogous to the zero sound mode in neutral Fermi liquid systems [12]. Still, for T=0, the value of $V_0 \Re e \Pi_{0R}$ is sufficiently close to unity at its maximum to produce a peak in the strength at $\omega = 14MeV$, as seen in figure 2. It should be noted that the position of this peak is in close agreement with the observed value (14 MeV). It is also interesting to note in figure 1 that as temperature increases the maximum in $\Re e \Pi_{0R}$ becomes weaker and weaker as a result of the averaging with the occupation numbers in equation 26. This leads to a broadening of the peak in the response function as can be checked in figure 2. By the time one reaches a temperature of 6 MeV only small collective effects remain visible.

The analysis we have just given also explains why some RPA calculations do give a rapid increase of the dipole width and some others do not. Indeed a slightly stronger residual interaction would be able to produce at zero temperature a sharp zero sound type mode which would disappear rapidly as temperature increases. In contrast a weaker residual interaction gives at zero temperature an already broad peak evolving slowly with temperature. An illustration of this point is given in the lower part of figure 2, which shows the evolution of the dipole mode for a residual interaction with $V_0 = 100$ MeV $\times$ fm^3. Similar results are obtained for lighter nuclei i.e. larger values of the momentum transfer q. One difference however is that the maximum of the real part of the response function becomes less pronounced as q increases which produces broader resonances. Of course the discussion we have given ignores finite size or surface effects and is further limited to the RPA framework. Some effects not included in this framework, such as the coupling to two particle-two hole states, are known to be important to describe the damping of collective vibrations [14]. Our analysis was further based on the Steinwedel and Jenssen model. We believe that analogous results would be obtained with the Goldhaber Teller model [10]. Indeed we expect in this case the response function to be dominated by the $q = \pi/2R$ mode with small corrections only arising from the multiples $q = n\pi/2R$ $(n > 1)$, because the peak in $\Re e \Pi_{0R}$ is much smaller for $n > 1$.

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Figure Captions

Figure 1 Real part of the non perturbed Lindhard function multiplied by $\hbar c$ (in fm^{-2}) as a function of the energy ω (in MeV) for a momentum q=.23 fm^{-1} and for different values of the temperature T=0,2,4 and 6 MeV.

Figure 2 Distribution of strength per unit volume for the operator $\exp(i\mathbf{q}.\mathbf{r})$ (in MeV$^{-1}\times$ fm^{-3}) as a function of the energy ω (in MeV) for a momentum q=.23 fm^{-1} and for different values of the temperature T=0,2,4 and 6 MeV. The upper curves correspond to an interaction strength V_0= 203 MeV$\times$fm^3 while the lower curve corresponds to V_0= 100 MeV $\times$ fm^3.